$$R^2 = Me^{j\phi}. \quad \text{(A-8)}$$

Substituting (A-8) into (A-6) and (A-7), each of those eigenvalue equations are transformed into two simultaneous equations for u' and u'', and these can be shown independently of modes as follows:

$$M = (u'^2 + u''^2) \cdot \sqrt{m_r^2 + m_i^2} \quad \text{(A-9)}$$

$$\phi = \tan^{-1}\left(\frac{2u'u''}{u'^2 + u''^2}\right) + \tan^{-1}\left(\frac{m_i}{m_r}\right) \quad \text{(A-10)}$$

where m_r and m_i are given as follows:

$$m_r = \{(\cos 2u' \pm \cosh 2u'')^2 + (\eta'^2 - \eta''^2)(\sin^2 2u' - \sinh^2 2u'') \mp 4\eta'\eta'' \sin 2u' \sinh 2u''\} / (\cos 2u' \pm \cosh 2u'')^2 \quad \text{(A-11)}$$

$$m_i = 2\{\eta'\eta''(\sin^2 2u' - \sinh^2 2u'') \pm (\eta'^2 - \eta''^2) \sin 2u' \sinh 2u''\} / (\cos 2u' \pm \cosh 2u'')^2. \quad \text{(A-12)}$$

The upper sign holds for the symmetric modes, while the lower sign holds for the antisymmetric modes. Simultaneous equations (A-9) and (A-10) may be solved graphically by plotting both contours of equal magnitude and argument, as shown in [11].

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The Edge-Guided Mode Nonreciprocal Phase Shifter

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***Abstract*—Results of a theoretical study are presented for a four-region model of a nonreciprocal dielectric–ferrite loaded stripline phase shifter employing the edge-guided dynamic mode. The behavior of our model in terms of the differential phase shift, the insertion loss, and the effective bandwidth is explored as a function of the various parameters involved.**

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I. Introduction

ONE OF THE fundamental structures which support the edge-guided mode is a ferrite loaded stripline with the dc magnetic biasing field oriented normal to the ground plane. This geometry was introduced by Hines [1] who explored the possibilities of obtaining broad-band nonreciprocal isolation and phase shifting. These two

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problems have been subsequently studied by a number of authors [2]–[5]. The structure supports a multiplicity of modes with the primary one being the field-displaced dynamic mode which relates directly to the no-cutoff mode for the isotropically loaded stripline. In addition, a magnetostatic mode is observed in the range of frequencies where the effective permeability is negative, but it suffers field displacement in the opposite sense to that of the dynamic mode. At lower frequencies volume modes occur which have essentially symmetric transverse field distributions as do the higher order modes. With a lossy region at or near one edge of the central conductor, the volume and magnetostatic modes as well as the higher order modes suffer substantial loss, and the useful bandwidth of the dynamic mode is limited only to the region over which it displays substantial field displacement. This turns out to be well in excess of the range of frequencies for which the effective permeability of the ferrite is negative, and has led to nonreciprocal isolator designs having bandwidths in the vicinity of two octaves. It is evident that the same structure will yield a device which exhibits nonreciprocal phase shifting if the lossy region loading one edge is replaced by a dielectric slab of high permittivity. The dynamic mode, for one direction of propagation, will adhere to this edge and will sense an effective dielectric constant much higher than would be the case for the reverse direction of propagation when it will be field displaced to the unloaded edge. Also the phase shift introduced into the signal path can be controlled by reversal of the dc biasing magnetic field since such reversal leads to a change of the preferred edge to which the dynamic mode will "cling."

Hines [1] has discussed a theoretical model for a stripline phase shifter in which the regions adjacent to the ferrite slab under the conducting strip (i.e., air on one side and a high-dielectric region on the other), are increased in width by an amount which is estimated will account for the effect of the fringing fields. These regions are then terminated by a magnetic wall.

Courtois *et al.* [5], on the other hand, have published some experimental and theoretical results obtained using a parallel-plate model containing up to two ferrite regions terminated on one side by a dielectric region and on the other by a magnetic wall, i.e., an infinite impedance boundary condition.

In this paper we present the results of a theoretical study of the differential phase shift and insertion loss for a parallel-plate model with a lossy ferrite region loaded on one side by a dielectric slab of high permittivity, while on the opposite side of the ferrite slab we have free space or regions of relatively low permittivity (Fig. 1). This geometry thus models the same structure as that of Hines. However, the two models differ in that the latter is closed by magnetic walls perpendicular to the ground plane. In contrast, our model extends to $\pm\infty$ in the transverse direction. A correction that would account for the fringing fields is not necessary since, as will be seen later, maxi-

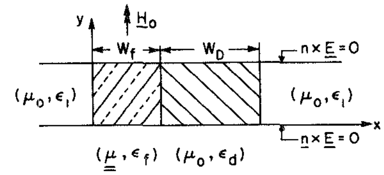

Fig. 1. Model geometry.

mum bandwidth for this device indicates the use of rather narrow ferrite regions. The center conductor will then generally extend beyond the ferrite region and into the dielectric regions, so that only a small fraction of the energy will be carried near the strip edges as a result of the rapid exponential decay of the fields away from the ferrite–dielectric interfaces. Hence, the fringing effect, in this case, cannot be significant.

It is our primary objective to examine the differential phase shift, the insertion loss and the effective bandwidth as a function of the various parameters involved. We also examine those factors which contribute to as constant a differential phase shift and insertion loss as possible over the widest possible bandwidth.

Data for several sample cases are presented as well as some information on characteristic impedance (Fig. 5).

II. Theory

We analyze this structure for the family of modes in which there is no variation of the field components in the direction of the applied magnetizing field. The fields are required to decrease exponentially away from the ferrite and dielectric slabs. This condition, together with the application of boundary conditions at all the interfaces, yields the following dispersion relation:

$$\tan k_f W_f - \frac{D_1 \sin k_d W_d + jD_2 \cos k_d W_d}{jD_3 \sin k_d W_d + D_4 \cos k_d W_d} = 0$$

where W_d and W_f are the width of the dielectric and ferrite regions, respectively, and

$$D_1 = k_f \mu_e (k_d^2 + k_1^2)$$
$$D_2 = 2k_f \mu_e k_1$$
$$D_3 = (k_1 \mu_e + \gamma\kappa')(k_d \mu_e - k_1 \gamma\kappa'/k_d) + k_1 k_f^2/k_d$$
$$D_4 = -(k_1 \mu_e + \gamma\kappa')(k_1 \mu_e - \gamma\kappa') - k_f^2$$

where

$$\mu_e = 1 + \chi - (\kappa^2/1+\chi), \qquad \kappa' = \kappa/(1+\chi)$$
$$k_1^2 = \gamma^2 + \omega^2 \mu_0 \epsilon_0 \epsilon_1$$
$$k_f^2 = \gamma^2 + \omega^2 \mu_0 \epsilon_0 \epsilon_f \mu_e$$
$$k_d^2 = \gamma^2 + \omega^2 \mu_0 \epsilon_0 \epsilon_d$$

and we have taken a field dependence of the form $e^{j\omega t - \gamma z}$. The ferrite is allowed to be lossy and this is taken into account in the constitutive parameters χ and κ by the modification $\omega_0=\gamma_0(H_0+j\Delta H_0/2)$, with γ_0 the gyromagnetic ratio, H_0 the dc magnetizing field, and ΔH_0 the ferrite resonance linewidth. To obtain the desired data we solve the transcendental equation for γ as a function of ω for both forward and reverse directions of propagation, i.e., for Im $\{\gamma\}\lessgtr 0$. This yields the phase shift per unit length as well as the insertion loss per unit length as functions of frequency for $\beta>0$ and $\beta<0$. The differential phase shift in degrees per centimeter is readily obtained as well as the insertion loss in decibels per centimeter.

III. Results

It is our objective to show the effect of the different parameters involved on the characteristics of a nonreciprocal phase shifter. Calculations were mainly performed for a particular ferrite material ($4\pi M_s=1780$ G, $\Delta H_0=45$ Oe, and $\epsilon_f=15$) and a fixed dc magnetizing field ($H_0=200$ Oe). Towards the end of this section comments are made regarding the effect of changing these parameters.

The dispersion characteristics were examined as a function of the width of the ferrite and the width and relative permittivity of the dielectric region. Our attention was focused on the differential phase shift, the insertion loss, and the effective bandwidth. The latter is limited by the onset of the first higher order mode and below by the magnetostatic surface mode.

The operation of the device depends on electrical asymmetry which is attained through structural asymmetry. This means that for one direction of propagation most of the energy is carried by the high-dielectric constant region through the field displacement effect. For the other direction of propagation the energy would tend to concentrate in the vicinity of the ferrite interface opposite to the dielectric region. However, due to the latter's higher permittivity, a greater part of the energy will be carried by the ferrite, resulting in a somewhat greater insertion loss. We cannot then attain equal insertion loss for both directions unless the loss of the dielectric material is carefully adjusted to yield such loss symmetry. At best a close match can be attained over only part of the operating band.

In maximizing the differential phase shift it is evident that maximal asymmetry is required. That is, as the relative permittivity of the dielectric region loading one side of the ferrite is increased so will the differential phase shift. However, this will also result in a decrease of the cutoff frequency of the first higher order mode and thus a reduction in bandwidth if the higher order mode is not suppressed. A reduction in the width of the ferrite region allows a decrease in the loss per unit length but will also result in a decrease in the differential phase shift while, at the same time, yielding a broader operating bandwidth.

The range of values of the parameters for which these various device attributes were examined were 1) width of the ferrite region from 1 to 10 mm, 2) width and relative dielectric constant of the dielectric region from 0.1 to 5 mm, and 3) 4 to 96, respectively. Some results illustrating the device performance as a function of the permittivity of the dielectric region are shown in Figs. 2 and 3. It is evident that increasing the dielectric loading increased the differential phase shift but substantially reduced the bandwidth of the device, while at the same time increasing the insertion loss.

Inspection of the field distribution confirms that at the low end of the band, for the case $\epsilon_d=96$ where the field displacement effect is maximum, the dominant part of the energy, when propagating in the reverse direction, is concentrated at the left-hand side of the ferrite slab with a substantial fraction carried by the lossless region where $\epsilon_1=1$. As the frequency is allowed to increase the effective permeability is less negative and the field displacement is weaker, resulting in a greater part of the reverse propagating energy being contained within the ferrite and high-dielectric regions. As a result, the differential phase shift

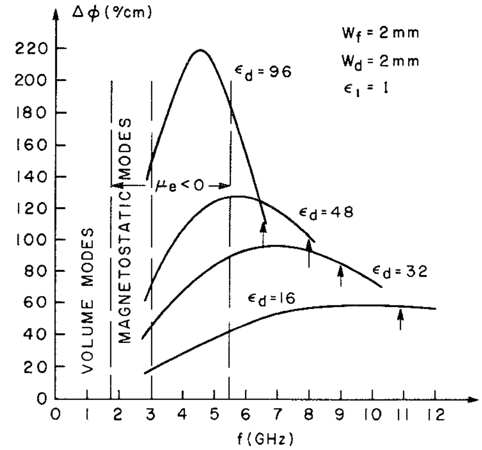

Fig. 2. Differential phase shift as a function of frequency for a ferrite material of $4\pi M_s=1780$ G, $\Delta H_0=45$ Oe, $\epsilon_f=15$, with $H_0=200$ Oe, ϵ_d variable, and $\epsilon_1=1$; Ferrite width 2 mm, dielectric width 2 mm, arrows indicate onset of first higher order mode.

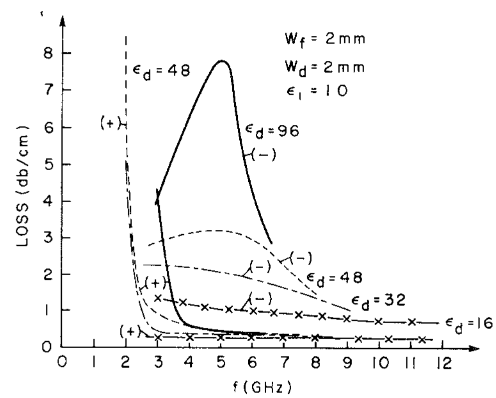

Fig. 3. Insertion loss as a function of frequency. (Parameters as for Fig. 1.) —— $\epsilon_d=96$; -------- $\epsilon_d=48$; -·-·-·-· $\epsilon_d=32$; x–x–x–x $\epsilon_d=16$. (−) and (+) indicate direction of propagation associated with insertion-loss characteristic.

decreases and the insertion loss increases at first and then, at still higher frequencies, as the dielectric region becomes dominant and most of the energy is now carried there, losses decrease rapidly as does the differential phase shift. For dielectrics with lower permittivity the same effect is observed but is moderated. When we reach a permittivity of the dielectric about equal to that of the ferrite the losses are almost uniform over the band while the differential phase shift varies little over almost an octave. Thus decreasing the loading to $\epsilon_d = 16$ will yield a 60-degree differential phase shift per centimeter with deviations of approximately ± 2 degrees over the entire X band, and with an associated insertion loss of less than 0.05 dB/cm in the forward direction and less than 0.1 dB/cm for the reverse direction. Note, however, that the onset of the higher order mode occurs at 10.9 GHz.

The possibility of obtaining an essentially constant differential phase shift and insertion loss over a given bandwidth is further explored. In Fig. 4 the width of the ferrite is halved, while the width of the dielectric region is also reduced so as to increase the cutoff of the higher order modes. We observe that for a relative dielectric constant of 32, the cutoff is well above X band and a differential phase shift of 73 degrees ($\pm 3°$) gives us an operating band from approximately 8 to 13 GHz. Allowing $\pm 5°$ phase deviations gives a bandwidth from 7.5 to 14 GHz or almost an octave. The insertion loss in the forward direction increases from 0.02 dB/cm to about 0.13 dB/cm, and for the reverse direction it decreases monotonically from a high of 0.10 dB/cm at the low-frequency end of the band to about 0.04 dB/cm. Decreasing the relative dielectric constant to 24 causes the differential phase shift to drop to 60°/cm ($\pm 5°$) over the X and Ku bands and leads to insertion losses of less than 0.03 dB/cm in the forward, and 0.07 dB/cm in the reverse direction. Suppression of the next higher order mode would extend operation into the K band and yields an octave and a half of bandwidth for an allowed phase deviation of $\pm 5°$/cm.

For these last two cases, the results of calculations of the characteristic impedance for the band of interest are presented. Also, an estimate will be made of the width that an actual microstrip of stripline phase shifter would have according to our model.

Fig. 5 gives the results of our impedance calculations for the range of interest (Fig. 4) and for $d = 0.65$ mm. We observe, first, that for both cases the impedance remains relatively constant over the band. For the worst case ($\beta < 0$, $\epsilon_D = 32$), the impedance varies ± 20 percent about the average value. Note also that since the fields are independent of variations in the vertical coordinate the impedance varies linearly with the thickness d.

The necessary transverse dimension of an actual device, as modeled by Fig. 1, can be indicated if we assume that the center conductor of the stripline geometry covers the ferrite and a sufficient part of the dielectric region so that little energy will be supported beyond those regions. The

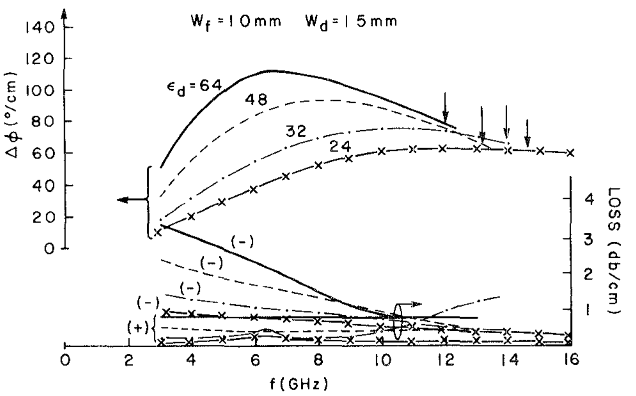

Fig. 4. Differential phase shift and insertion loss for reduced width ferrite and dielectric sections. Electrical parameters as for Fig. 1. —— $\epsilon_d = 64$; --------- $\epsilon_d = 48$; -·-·-·-·- $\epsilon_d = 32$.

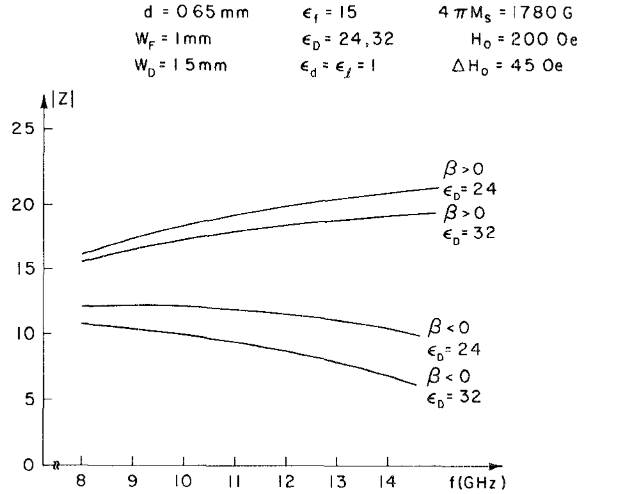

Fig. 5. Modal impedance as a function of frequency. Parameters as for Fig. 4.

field intensities will then be small at the edges of the conductor. An estimate of the required width of the center conductor is given by the distance (in the transverse direction) between those points for which the electric field amplitude is 10 percent of its maximum value at the ferrite–dielectric interface. For the two cases under consideration, this yields a maximum width of 10 mm for a band from 8 to 14 GHz. This value is in substantial agreement with that given by Hines in [1] for his stripline phase shifter with a ferrite slab of 7.5-mm width and an effective bandwidth extending over 1 to 4 GHz.

Finally, the effect of choosing a different ferrite material and of modifying the biasing magnetic-field values from those used in this study is considered. First, it is obvious that it is always advantageous to use a low linewidth ferrite for minimizing the insertion loss. Second, a lower saturation magnetization and dc magnetizing field than that used here will lower the negative effective permeability range and hence, the onset of the magnetostatic surface mode. Thus by keeping the widths of the ferrite and dielectric slabs at conveniently small values, so as to

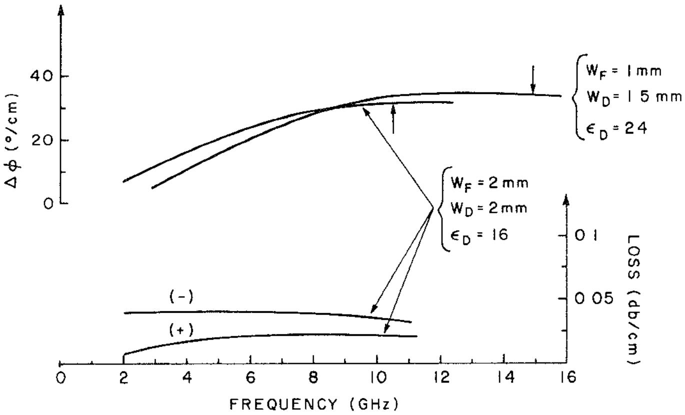

Fig. 6. Differential phase shift and insertion loss for a device with a ferrite material of $4\pi M_s = 1000$ G, $\Delta H_0 = 45$ Oe, $\epsilon_f = 15$, and $H_0 = 100$ Oe.

delay the onset of the higher order modes, a wider operating bandwidth can be obtained at the cost of reducing the differential phase shift per centimeter. As an example of this, we include the results (Fig. 6) for two further cases where the parameters are essentially the same as before but where the material has been changed to one having a saturation magnetization of $4\pi M_s = 1000$ G. We find a differential phase shift ranging from 18°/cm at 5 GHz to 35°/cm at 15 GHz (where the onset of the first higher order mode occurs). This result indicates that the device designed by Courtois *et al.* in [5] seems to approach the performance we predict in the differential phase shift attained but, at the same time, further reductions in insertion loss should be possible.

IV. Conclusions

An analysis and numerical study of a nonreciprocal phase shifter employing the field displaced dynamic mode, i.e., edge-guided mode, was performed on a model of a ferrite–dielectric loaded stripline. Results obtained for the differential phase shift and insertion loss per unit length for a wide range of electrical and dimensional parameters indicated that for given ferrite material, differential phase shifts of the order of 60°/cm could be obtained over bandwidths approaching one and one-half octaves and with insertion losses of the order of 0.2 to 0.1 dB/cm. The permittivity of the material loading one side of the ferrite insert dominated in controlling the differential phase shift obtained. Increasing the relative dielectric constant increased the differential phase shift obtained but decreased the effective bandwidth for given allowable phase deviation and increased the effective loss.

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